

Spectral Emissivity of Silicon

Tsutomu SATO

*The Research Institute for Scientific Measurements
Tohoku University, Gojyo-Dori, Sendai*

(Received November 2, 1966)

The emissivity of silicon was observed in the spectral region from 0.4 to 15 μ at various temperatures from 340°K to 1070°K by using two n-type specimens with the resistivity of 15 ohm-cm and 7×10^{-3} ohm-cm at 300°K, respectively. The thermal radiation of silicon consists of three types due to band-to-band transition, free carriers and lattice vibration. It was made clear from the emissivity measurement that, in a pure specimen, the radiation due to lattice vibration is conspicuous at low temperatures, while that due to free carriers is predominant at higher temperatures, and that, in a doped specimen, the radiation due to free carriers is predominant even at low temperatures. The observed emissivity is close to the theoretical formula represented by the refractive index n and the extinction coefficient k . The temperature dependence of n and k is shown and a brief discussion is also added.

§ 1. Introduction

The emissivity is a physical quantity not only basic in the theory of thermal radiation but also important in the practice of optical pyrometry. When the radiant heat transfer is in an equilibrium state, the emissivity of perfectly opaque bodies is given by Kirchhoff's law as $1-R$ where R is the reflectivity. McMahon¹⁾ generalized this law to apply to partially transparent bodies. If the multiple internal reflections are taken into consideration, the emissivity ϵ of a plane-parallel specimen is expressed for normal incidence by the following equation

$$\epsilon = \frac{(1-R)(1-T_r)}{1-R \cdot T_r}, \quad (1)$$

where R is the true reflectivity which is given by

$$R = \frac{(n-1)^2 + k^2}{(n+1)^2 + k^2},$$

while T_r is the true transmissivity and is given by

$$T_r = \exp(-Kt) = \exp(-4\pi kt/\lambda),$$

where

n =refractive index,

k =extinction coefficient,

K =absorption coefficient,

λ =wavelength,

t =thickness of the specimen.

When Kt is large and $T_r \approx 0$, the emissivity becomes

$$\epsilon = 1 - R = \frac{4n}{(n+1)^2 + k^2}. \quad (2)$$

On the other hand, when Kt is small and $1-R/R \gg Kt$, the emissivity becomes

$$\epsilon \approx Kt. \quad (3)$$

From the above equations, it is seen that the emissivity is closely related to the optical constants. Therefore, by measuring the emissivity, the optical behaviour of the specimen can be made clear especially in higher temperature region.

If the apparent reflectivity and transmissivity are denoted by R^* and T_r^* , the following relation holds:

$$\epsilon + R^* + T_r^* = 1, \quad (4)$$

where

$$R^* = R \left\{ 1 + \frac{T_r^2(1-R)^2}{1-R^2 \cdot T_r^2} \right\},$$

and

$$T_r^* = T_r \frac{(1-R)^2}{1-R^2 \cdot T_r^2}.$$

The equation (4) indicates that the emissivity can be obtained by an indirect method of measuring the reflectivity and transmissivity of the specimen as well as by a direct method of comparing the radiation of the specimen with that of a blackbody.

A considerable number of experimental studies have hitherto been reported on the emissivity of various metals and refractories; most of them are measurements at the wave-

length of $0.65\ \mu$ mainly for the use in optical pyrometry, and those which cover some spectral region are not so many. The observations on the emissivity of semiconductors^{2,3)} are quite limited in number. As to silicon, Allen⁴⁾ measured the emissivity at various temperatures but the wavelength concerned was restricted to $0.65\ \mu$. Recently, Stierwalt, Bernstein and Kirk⁵⁾ tried to observe emissivity at comparatively low temperatures and obtained the radiation spectrum due to lattice vibration in the infrared region. At the temperatures above 500°K , the observation of the spectral emissivity has scarcely been reported.

In the case of silicon, the absorption due to band-to-band transition, to free carriers and to lattice vibration lies in the visible and infrared region; hence, at temperatures up to the melting point, the thermal radiation arising from different origins are emitted. In the present work, the emissivity of silicon was observed in the spectral region from 0.4 to $15\ \mu$ at various temperatures up to 1070°K using both direct and indirect methods, in order to obtain some information on the radiant behaviour and its correlation with the optical constants. The detail of the experimental procedure and the results obtained will be described in the following.

§ 2. Experimental

Two kinds of n-type silicon crystals doped with phosphor, were used in this experiment: one was comparatively pure with the resistivity of $15\ \text{ohm-cm}$, and the other, heavily doped to have the resistivity of about $7 \times 10^{-3}\ \text{ohm-cm}$ at 300°K . The two specimens mainly used were cut from the former crystal into disks $23\ \text{mm}$ in diameter and 1.77 and $3.71\ \text{mm}$ in thickness respectively. Besides these, a specimen was cut from the latter crystal into a disk $23\ \text{mm}$ in diameter and $0.2\ \text{mm}$ in thickness. The specimens were prepared for use by grinding and polishing them in plane-parallel according to metallographic techniques and subsequently by optical techniques.

In the direct measurement of emissivity, the emission of the silicon specimen must be compared with that of the blackbody at the same temperature. As a blackbody radiation source, the V-shape cavity of graphite with

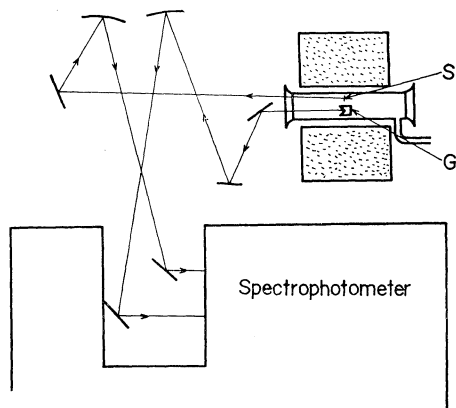


Fig. 1. Optical arrangement for the direct measurement of emissivity. G, V-shape cavity of graphite; S, specimen.

the emissivity of about 0.97 was employed. Figure 1 shows an optical arrangement used in the measurement. The specimen and the cavity are inserted in a silica tube provided with NaCl windows at both ends and are placed at the centre of an electric furnace.

The radiations from the silicon and from the cavity are introduced to the optical paths of the sample and reference beams in a Japan Spectroscopic IR-S spectrophotometer fitted with NaCl prism, which covers the spectral region from 2.5 to $15\ \mu$. In this spectrophotometer, the number of reflection between sample chamber and entrance slit is different by one for two beams; this is the reason why the number of reflecting mirrors in the figure is different for two radiations.

As the specimen is enclosed in a furnace in this case, care must be taken so that the thermal radiation of the specimen will not be contaminated by that of the silica tube or of the wall of the furnace. Therefore, in practice, the surface of the specimen was adjusted carefully to be normal to the optical path accurately. This, together with excellent optical quality of the specimen surface reduced the stray light to a negligible quantity. The temperature of the furnace was controlled by a power proportioning electronic regulator to within $\pm 1^\circ\text{K}$. At lower temperatures, fine regulation by hand was also used to obtain higher stability.

For wavelengths shorter than $2.5\ \mu$, emissivity measurement was made by a double pass spectrophotometer fitted with LiF prism which was designed and constructed in our

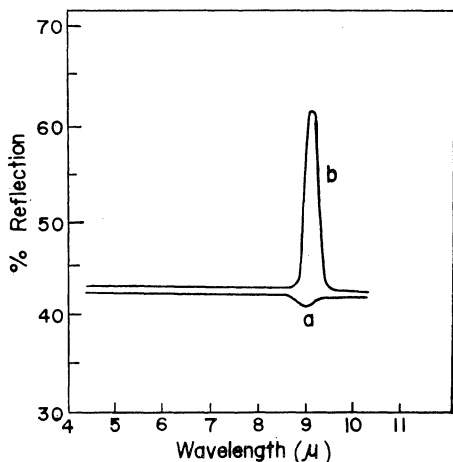


Fig. 2. Reflectivity of silicon before (a) and after (b) heating in the atmosphere.

Institute. In this case, the radiations from the specimen and from the cavity were introduced into the entrance slit alternately and the output voltages of the detector were compared. The relation between the intensity of input radiation and the output voltage was calibrated by using a comb attenuator.

Besides the direct method already mentioned, the indirect method was been also used of measuring the apparent transmissivity and reflectivity. To measure the transmissivity, the graphite cavity in Fig. 1 was removed and the beam from a light source was passed through the silica tube and was introduced

to the entrance slit of a single beam spectrophotometer. A spectrophotometer of double pass type, mentioned in the above, and Hitachi EPU-2 were used for this purpose. By placing the specimen in and out of the beam within the tube, output voltages of the detector were compared.

In measuring the reflectivity, the radiation from a light source was divided into two beams: one served as a reference beam, and the other was reflected by the specimen. The incident angle to the specimen was about 4° . As the standard of reflectivity, an aluminized mirror with known spectral reflectivity was used. In both measurements, the beam incident upon the specimen was chopped at the frequency of 10 c/s; by this, the effect of thermal radiation from the specimen at elevated temperatures was eliminated thoroughly.

In the presence of oxygen, the silicon surface is oxidized rapidly at elevated temperatures, which can be detected by a Reststrahlen of Si-O at 9μ . In Fig. 2, the reflectivities of silicon disk before (a) and after (b) the test heating in the atmosphere are presented. A faint absorption at 9μ in (a) is an effect of oxygen in the crystal occluded during the crystal growth on the beams reflected by back surface. Throughout the measurements, the specimens were held in vacuum of about 10^{-4} mm Hg; by this, the specimen surfaces

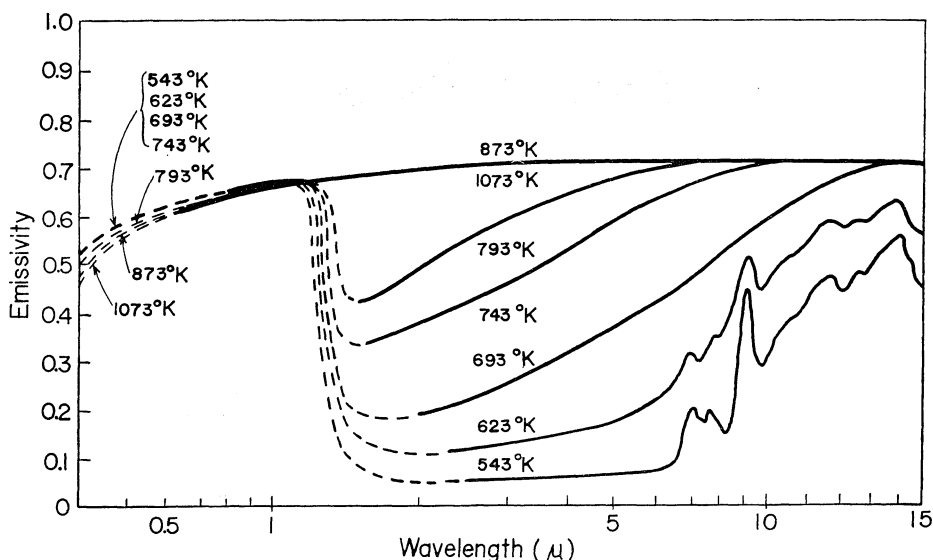


Fig. 3. Spectral emissivity of pure specimen. Solid curves: direct measurement. Dotted curves: indirect measurement.

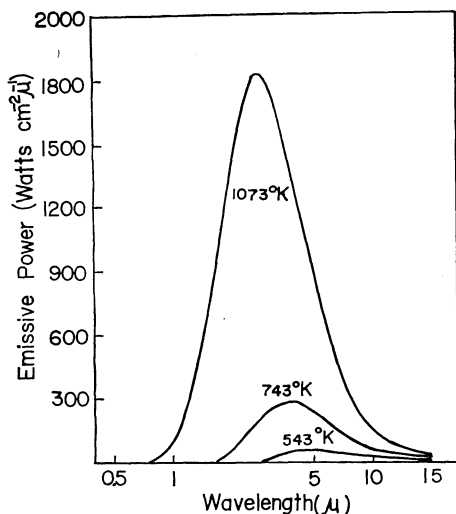


Fig. 4. Spectral emissive power of blackbody.

could be maintained free from oxidation.

§ 3. Results Obtained

3.1. General aspect of emissivity

Using the direct and indirect methods, the spectral emissivities were measured at various temperatures; the results obtained at 543, 623, 693, 743, 793, 873, and 1073°K using the pure specimen with the thickness of 1.77 mm are shown in Fig. 3. In this figure, the solid curves are the values obtained by the direct method while dotted curves are those by the indirect one. The solid curves are not given in shorter wavelengths at lower temperatures because the thermal radiation is too faint to be measured in this region. For convenience sake, the blackbody radiation curves at corresponding temperatures are shown in Fig. 4. Again in Fig. 3, the values obtained by two methods show good consistency, which implies that the equilibrium of the energy transfer between the specimen and the furnace was kept for individual wavelength.

In the visible region, the emissivity is fairly high, since the direct and indirect of band-to-band transitions occur at this spectral region. At low temperatures, the specimen is transparent for the wavelengths longer than 1.2 μ , hence the emissivity is very low in near infrared. For the wavelengths longer than 6 μ , however, comparatively high emissivity due to lattice vibration appears. As this specimen becomes intrinsic at about 570°K, the emissivity by free carriers can be

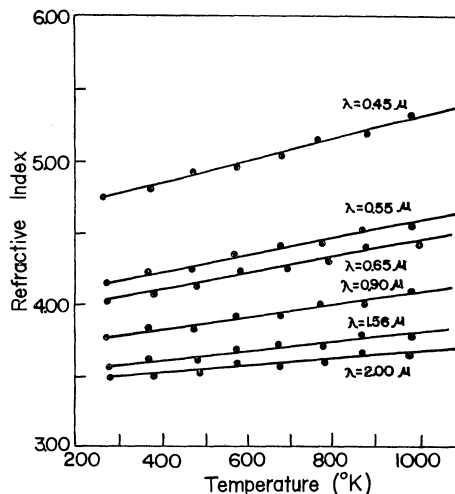


Fig. 5. Temperature dependence of the refractive index.

seen in near infrared at higher temperatures. It enhances rapidly with temperature and finally masks the emissivity by lattice vibration. At the temperatures higher than 870°K, the emissivity curve becomes nearly straight in the whole infrared region. In the following, the emissivity arising from three different origins will be described in detail.

3.2. Emissivity due to band-to-band transition

For the wavelengths shorter than 1.2 μ , the specimen is opaque perfectly, hence the equation (2) previously given may be applicable. As has been reported by Dash and Newman,⁶⁾ the value of k is about 0.22 at 0.4 μ and rapidly decreases with increasing wavelengths; hence the relation $n \gg k$ exists in this case. Therefore the equation (2) may safely be substituted by

$$\epsilon \approx \frac{4n}{(n+1)^2}. \quad (5)$$

As n increases with decreasing wavelength, the emissivity decreases as is seen in Fig. 3. By raising temperature, the emissivity decreases with a small factor. At 1070°K, the emissivity for 0.65 μ becomes 0.60 which is slightly smaller than the value obtained by Allen.⁷⁾ This discrepancy may be ascribed to the difference of the optical quality of the specimen surface, since his specimen was not optically polished.

Using the equation (5), the value of n can be obtained from the observed emissivity. The value at low temperature thus calculated

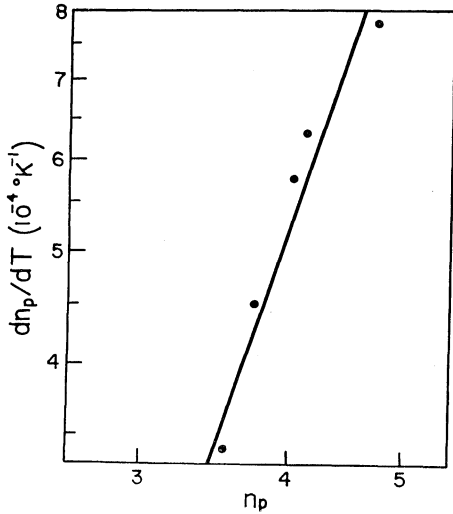


Fig. 6. Variation of dn_p/dT with refractive index.

shows a good accordance with that obtained by Philipp and Taft.⁹¹ As k is considered not to show large enhancement with temperature, the change of emissivity with temperature gives the temperature dependence of refractive index which is represented in Fig. 5. The refractive index of pure specimen n_p can be expressed by the equation

$$n_p^2 - k^2 = 1 + \frac{4\pi N e^2 (\omega_0^2 - \omega^2)}{m \{ (\omega_0^2 - \omega^2)^2 + g^2 \omega^2 \}},$$

where g is a damping constant and ω_0 is the frequency of the maximum intrinsic absorption. In the present case, the value of k is small and the variation of N with temperature is not large. Hence, for the frequency region where $(\omega_0^2 - \omega^2) \gg g\omega$, the above equation may be substituted by

$$n_p^2 = 1 + A(\omega_0^2 - \omega^2)^{-1}.$$

Therefore, the temperature dependence of n_p is given by

$$\begin{aligned} \frac{dn_p}{dT} &= \frac{dn_p}{d\omega_0} \frac{d\omega_0}{dT} \\ &= -\frac{\omega_0}{A} \frac{(n_p^2 - 1)^2}{n_p} \frac{d\omega_0}{dT} \approx -\frac{\omega_0}{A} n_p^3 \frac{d\omega_0}{dT}. \end{aligned} \quad (6)$$

As is pointed out by Lukes,⁹¹ $d\omega_0/dT$ is nearly constant and negative in sign, hence n_p increases linearly with temperature which is well represented by the lines in Fig. 5. In Fig. 6, the inclination of these lines dn_p/dT is plotted against n_p at the corresponding wavelength in logarithmic scale. The curve

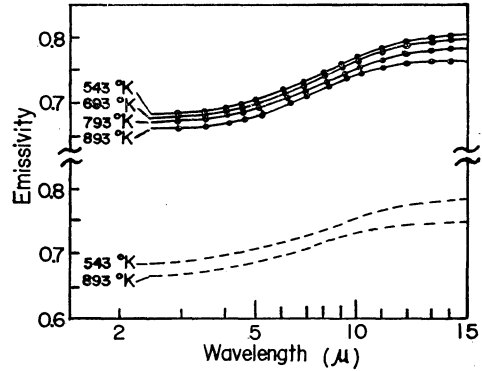


Fig. 7. Spectral emissivity of doped specimen. Solid curves: experimental. Dotted curves: calculated.

indicates that dn_p/dT is proportional to n_p^3 as is given by the equation (6).

3.3. Emissivity due to free carriers

3.3. a opaque specimen

The emissivity of free carrier radiation is governed by the impurity concentration of the crystal, and the thickness of the specimen as well as by the temperature at which the specimen is kept. In Fig. 7, the observed emissivity of the doped specimen with the resistivity of 7×10^{-3} ohm-cm is represented by solid curves. The specimen is practically opaque even at low temperatures and shows sufficiently high value of emissivity. In this case, the emissivity is given by the equation (2), in which n and k should be replaced by those for the doped specimen, n_d and k_d . Therefore, n_d and k_d can be approximately represented as follows:

$$n_d^2 - k_d^2 = n_p^2 - \frac{4\pi N_e e^2 / m_e}{\omega^2 + g_e^2} - \frac{4\pi N_h e^2 / m_h}{\omega^2 + g_h^2}, \quad (7a)$$

$$n_d k_d \omega = \frac{2\pi N_e e^2 g_e / m_e}{\omega^2 + g_e^2} + \frac{2\pi N_h e^2 g_h / m_h}{\omega^2 + g_h^2}, \quad (7b)$$

where N_e , m_e and g_e are concentration, effective mass and damping constant of electrons, and N_h , m_h and g_h are those of holes, respectively. The terms related to holes should be considered only at the highest temperature in the measurement. For the present carrier concentration, k_d^2 in the equation (7a) is still very small and may be neglected except in longer wavelengths. The value of n_p is nearly constant in the infrared region, but the value of $4\pi N_e e^2 / m_e (\omega^2 + g_e^2)$ increases with increasing wavelength; hence

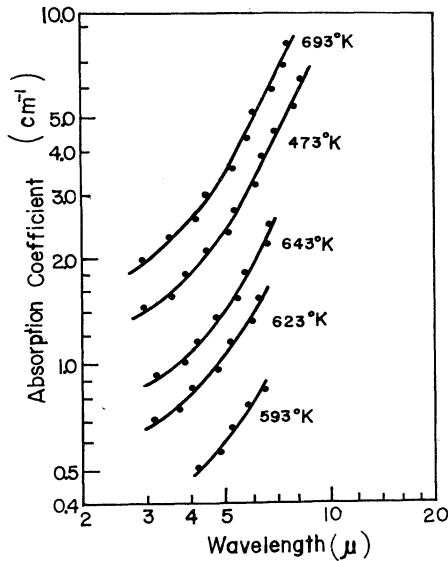


Fig. 8. Spectral dependence of the absorption due to free carriers.

the emissivity shows a small enhancement with wavelength. The temperature dependence of emissivity in this region is determined by that of n_p in the equation (6). For the wavelengths longer than 8μ , however, the value of k_d becomes a considerable amount which increases with increasing temperature; hence the emissivity shows larger decrease with temperature. Using the equation (7), the spectral emissivity at 543°K and 893°K has been calculated which is represented in dotted curves in Fig. 7. In this calculation, the values of m_e and m_h used are those given by Spitzer and Fan^{10,11} which are 0.3 m and 0.37 m at 543°K and 0.3 m and 0.37 m at 893°K , respectively. The value of N_e at 543°K has been obtained from the measured electrical resistivity using the data given by Backenstoss,¹² to be $8 \times 10^{18} \text{ cm}^{-3}$, while the value at 893°K , to be $8.5 \times 10^{18} \text{ cm}^{-3}$ which includes the intrinsic carrier concentration obtained using the relation given by Morin and Maita.¹³ At 893°K , the effect of holes has been also taken into account taking the value of N_h $5.2 \times 10^{17} \text{ cm}^{-3}$ given by Morin and Maita. The value of g_e at 543°K has been estimated using the data given by Backenstoss assuming that the impurity scattering mobility is large compared to the lattice scattering mobility to be $3.7 \times 10^{13} \text{ sec}^{-1}$. The values of g_e and g_h at 893°K have been

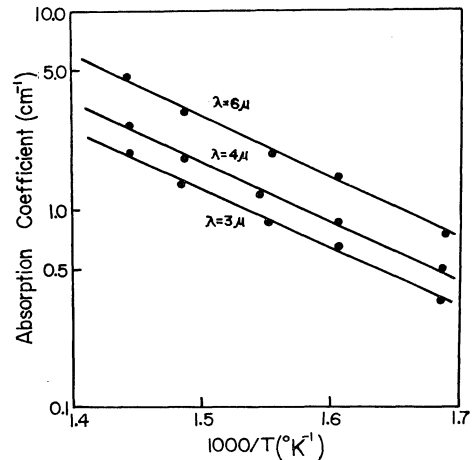


Fig. 9. Temperature dependence of the absorption due to free carriers.

obtained using the relation given by Morin and Maita, to be $6.9 \times 10^{13} \text{ sec}^{-1}$ and $1.2 \times 10^{14} \text{ sec}^{-1}$, respectively. In Fig. 7, the calculated curves will be seen to be in good accordance with the observed curves in the same figure.

3.3. b. partially transparent specimen

When a specimen of comparatively pure crystal has a small thickness to be more or less transparent, the emissivity ϵ_t may be represented by the equation (1). By the equation (2), R is given by $1 - \epsilon_\infty$ where ϵ_∞ is the emissivity of the same crystal with the thickness large enough to be practically opaque. Therefore, equation (1) can be replaced by

$$\epsilon_t = \frac{\epsilon_\infty(1 - T_r)}{1 - (1 - \epsilon_\infty)T_r}, \quad (8)$$

which gives the following relation

$$\exp(Kt) = 1 + \frac{\epsilon_\infty \epsilon_t}{\epsilon_\infty - \epsilon_t}.$$

Thus, by measuring the emissivity of thin and thick specimens, the absorption coefficient can be obtained. The spectral dependence and the temperature dependence of K obtained by the specimen with the resistivity of 15 ohm-cm are plotted in Figs. 8 and 9, respectively. From the equation (7b), it is expected that the frequency dependence is inversely proportional to ω^2 because g^2 is far smaller than ω^2 in this case. This relation is well satisfied in the curves in Fig. 8 for the wavelengths longer than 5μ especially at higher temperatures, but the departure from the quadratic relation is

seen in shorter wavelengths. It has been pointed out by Spitzer and Fan¹⁴⁾ that at temperatures where the lattice scattering is predominant, the absorption given by the equation (7b) should be multiplied by a factor $(4/9\sqrt{\pi})(\hbar\omega/kT)^{1/2} <(1+2E/\hbar\omega)(1+E/\hbar\omega)^{1/2}>_{av}$, where the averaging is done over all electron energies E . It leads to the fact that the frequency dependence is proportional to ω^{-2} in longer wavelengths but tends to $\omega^{-1.5}$ for short wavelengths where $\hbar\omega > kT$. Most of the departure mentioned above can be interpreted by this consideration. Another interpretation of the departure is that it is caused by the intraband transition in the conduction band. As has been observed by Spitzer and Fan, the transition gives an absorption extending from 1.5 to 5 μ superimposed to the free carrier absorption, which is proportional to the impurity concentration. The impurity concentration of the present specimen, however, seems too small to give the measured departure. Moreover, it has been reported that the absorption becomes faint at elevated temperatures, where the absorption by holes becomes comparable to that by electrons; hence, if it were responsible to the departure, the effect should be considered not to be prominent.

Since K is proportional to electrical conductivity, the temperature dependence of K should be given by that of conductivity which is proportional to $\exp(-E_g/2kT)$. The temper-

ature dependence given in Fig. 9 was found to be in excellent agreement with that calculated by using the measured conductivity and the effective mass of $m_e=0.3m$ given by Spitzer and Fan.

Therefore, if the electrical conductivity is known, the emissivity due to free carriers can be given by the equation (8) for a specimen with a given thickness, since ϵ_∞ is given by the equation (2).

3.4. Emissivity due to lattice vibration

Emission spectra due to the lattice vibration of silicon appear in the wavelength from 6 to 25 μ , of which those up to 15 μ were observed. Besides these, a strong lattice vibration of Si-O appeared at 9 μ , by the oxygen occluded during crystal growth. In this wavelength region, the emissivity due to lattice vibration is superimposed by that due to free carriers at elevated temperatures, as is seen in Fig. 3. The emissivity due to free carriers can be estimated by extrapolating the curves in Fig. 8 as K is known to be proportional to λ^2 . Therefore, the true emissivity due to lattice vibration can be obtained by subtracting this free carrier term from the measured value. The emissivity due to lattice vibration thus obtained is represented in Fig. 10. Starting from 343°K, the emissivity increases with increasing temperature, reaches maximum at 493°K and then decreases with further increasing temperature. The values of the emissivity peaks at 6.9 μ , and 10.4 μ at various temperature

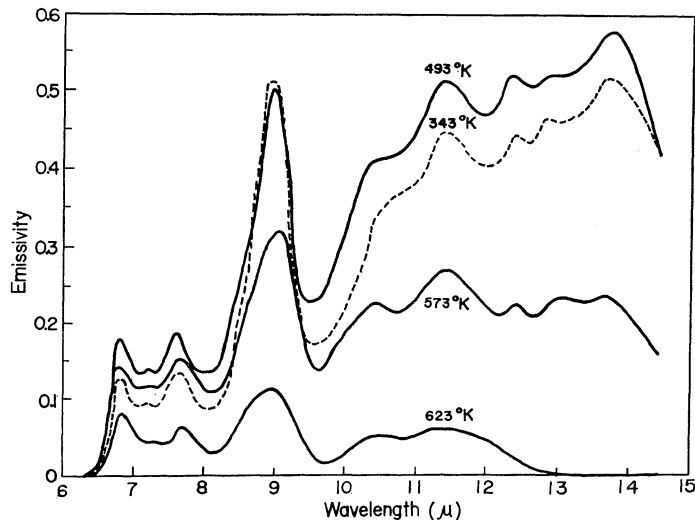


Fig. 10. Spectral emissivity due to lattice vibration.

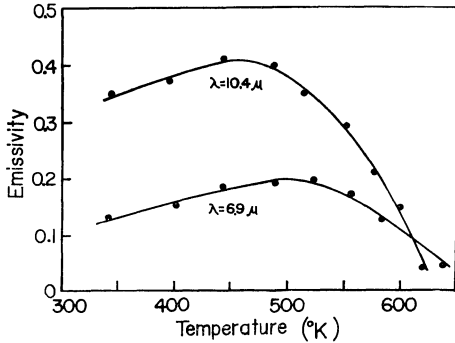


Fig. 11. Temperature dependence of the emissivity due to lattice vibration. Solid curves: calculated.

are plotted in Fig. 11. The absorption band due to the lattice vibration in silicon was observed by Collins and Fan¹⁶⁾ in the temperature range from 5 to 400°K. At higher temperatures, the absorption was found to increase linearly with temperature, to which they give an interpretation that a vibrational

$$\varepsilon_l = \varepsilon_{f+l} - \varepsilon_f = \frac{(1-R)^2 [\exp(-K_f \cdot t) - \exp(-K_{f+l} \cdot t)]}{(1-R \cdot \exp(-K_{f+l} \cdot t))(1-R \cdot \exp(-K_f \cdot t))}$$

where K_{f+l} is the absorption of both free carriers and lattice vibration and K_f is that of carriers which is approximately given by

$$K_f = 4\pi N_e e^2 g_e / c n m_e (\omega^2 + g_e^2) + 4\pi N_h e^2 g_h / c n m_h (\omega^2 + g_h^2)$$

Therefore, if the absorption by lattice vibration K_l is assumed to be proportional to T , the values of ε_l can be calculated which becomes as is represented by the curves in Fig. 11. The accordance between the calculated and the observed values is excellent which indicates that K_l is proportional to T up to the highest temperature.

3.5. Emissivity of a specimen heated by passing electric current

The results mentioned in the preceding were obtained by placing a specimen in an electric furnace. In this case, the furnace makes a cavity and the radiant energy with the spectral distribution very near to that of the blackbody radiation flows into the specimen. Some of the radiant energy will be absorbed within the specimen but the same amount of radiation is emitted; thus the radiant energy keeps equilibrium for every wavelength. When the specimen placed in free space is heated by passing electric cur-

rent through it, on the other hand, the electric field applied accelerates the electrons in conduction band, which makes the electrons in the band to be excited into higher states or atoms to induce lattice vibration. Thus the electric energy which flows into the specimen is transferred into heat energy which flows out of the specimen through heat conduction or radiation to reach thermal equilibrium finally. Therefore, it might happen that the spectral emissivity in such a case is somewhat different from that obtained by a specimen placed in a furnace. In this consideration, a narrow slab cut from a crystal with the resistivity of 15 ohm-cm was heated by passing alternating current and its emissivity was measured at 543°K and 573°K. The spectral emissivity in the infrared region, however, showed an excellent agreement with that obtained when a specimen is placed in a furnace, and no anomaly could be found in the relation between the emissivities due to free carriers and lattice vibration. From these results, it is considered that the energy of electrons in conduction band has a distribution very near to that of blackbody at the same temperature.

§ 4. Conclusion

The thermal radiation of semiconductors consists of three radiations due to band-to-band transition, free carriers and lattice vibrations, the proportion of which is governed by the band gap and lattice vibration frequency as well as the impurity concentration, thickness and temperature. At low temperatures at which silicon is extrinsic, most of the radiation is caused by lattice vibration, while at temperatures higher than 570°K at which silicon is intrinsic, the major part of radiation is that of free carriers. The radiation due to band-to-band transition appears only at high temperatures because the energy gap is large compared to the thermal energy. In the case of doped specimen, on the other hand, the radiation due to free carriers is prominent even at low temperatures.

In all cases, the spectral emissivity is well represented by the theoretical formula involving refractive index and extinction coefficient as parameters. Therefore, when the emissivity is needed for optical pyrometry, the values can be given provided the impurity concentration is known. At elevated temperatures, small change of the refractive index appears caused by the variation of both intrinsic and free carriers absorptions. Even for the impurity concentration as high as $8 \times 10^{18} \text{ cm}^{-3}$, the contribution of extinction coefficient to the emissivity is small; hence the emissivity obtained is far different from that of metals in feature.

The results obtained by a specimen heated by passing electric current shows that when the thermal equilibrium is reached within a specimen, the emissivity of thermal radiation depends on the temperature only and does

not on the method how it is heated. When the thermal equilibrium is not reached, the emissivity is expected to be different which, however, is a problem beyond the present investigation.

Acknowledgements

The writer would like to express his sincere thanks to Professor Takemaro Sakurai for his kind guidance, technical advice and continuous encouragement throughout the course of this work.

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